

例 41 求函数 $f(x) = \sin x - \ln(1+x)$ 的极值

1. 求导数 $f'(x) = \sin x - \frac{1}{1+x}$

2. 求驻点 $f'(x) = 0$ 的解

3. 求二阶导数 $f''(x)$

4. 判断 $f'(x)$ 的正负

$$f'(x) = \sin x - \frac{1}{1+x} \quad f''(x) = \cos x + \frac{1}{(1+x)^2}$$

$$f'(x) = \sin x - \frac{1}{1+x} \quad f''(x) = \cos x + \frac{2}{(1+x)^3} < 0 \quad (-1, \frac{\pi}{2})$$

$\therefore f'(x)$ 在 $(-1, \frac{\pi}{2})$ 上单调递减

$$f'(0) = 1 - \frac{1}{1+0} = 0 \quad f''(0) = \cos 0 + \frac{2}{(1+0)^3} = 3 > 0$$

$f'(x)$ 在 $(-1, \frac{\pi}{2})$ 上单调递减，且 $f'(0) = 0$ ，故 $f(x)$ 在 $x=0$ 处取得极大值

$f(x)$ 在 $(-\frac{\pi}{2}, 0)$ 上单调递增，在 $(0, \frac{\pi}{2})$ 上单调递减

2. 求 $f(x)$ 在 $x=0$ 处的函数值 $f(0) = 0$

3. 求 $f(x)$ 在 $x=0$ 处的二阶导数 $f''(0) = 3 > 0$

$$f(x) \text{ 在 } (x_0, \frac{\pi}{2}) \text{ 上单调递增，且 } f(x_0) > 0 \quad f''(\frac{\pi}{2}) = \cos \frac{\pi}{2} + \frac{2}{(1+\frac{\pi}{2})^3} < 0$$

4. 求 $f(x)$ 在 $x=\frac{\pi}{2}$ 处的函数值 $f(\frac{\pi}{2}) = 1 - \ln(1+\frac{\pi}{2})$

5. 求 $f(x)$ 在 $x=\frac{\pi}{2}$ 处的二阶导数 $f''(\frac{\pi}{2}) = \cos \frac{\pi}{2} + \frac{2}{(1+\frac{\pi}{2})^3} < 0$

$$\square \quad x \in (x_1, \frac{\pi}{2}) \quad \square \square \quad f(x) \quad \square \square \square \square \quad f(x) < f(x_1) = 0 \quad \square \quad f(x) \quad \square \square \square \square$$

$$\square \quad x \in (\frac{\pi}{2}, \pi) \quad \square \square \quad \cos x < 0 \quad \square \quad -\frac{1}{1+x} < 0 \quad \square \square \quad f(x) = \cos x - \frac{1}{1+x} < 0 \quad \square \quad f(x) \quad \square \square \square \square$$

$$\square \square \quad f(\frac{\pi}{2}) = 1 - \ln(1 + \frac{\pi}{2}) > 1 - \ln(1 + \frac{3.2}{2}) = 1 - \ln 2.6 > 1 - \ln e = 0 \quad \square$$

$$f(\pi) = -\ln(1 + \pi) < -\ln 3 < 0 \quad \square$$

□□□□□□

x	$(-1, 0)$	0	$(0, x_1)$	x_1	$(x_1, \frac{\pi}{2})$	$\frac{\pi}{2}$	$(\frac{\pi}{2}, \pi)$	π
$f(x)$	-	0	+	0	-	-	-	-
$f'(x)$	□□□□	0	□□□□	□□ 0	□□□□	□□ 0	□□□□	□□ 0

$$\square \square \square \square \square \square \square \square \quad f(x) \quad \square \quad (-1, \frac{\pi}{2}] \quad \square \square \square \square \square \square \square \square \quad 0 \quad \square$$

$$\square \square \square \square \square \square \square \square \square \square \quad f(x) \quad \square \quad (\frac{\pi}{2}, \pi) \quad \square \square \square \square \square \square \square \square \quad x_1 \quad \square$$

$$\square \quad x \in [\pi, +\infty) \quad \square \square \quad \sin x, 1 < \ln(1+x) \quad \square \square \quad f(x) = \sin x - \ln(1+x) < 0 \quad \square \square \square \square$$

$$\square \square \square \quad f(x) \quad \square \quad [\pi, +\infty) \quad \square \square \square \square$$

$$\square \square \quad f(x) \quad \square \square \square \square 2 \quad \square \square \square \square$$

$$2 \square \square \square \square \quad f(x) = \ln x - x + 2 \sin x \quad \square \square \square \square$$

$$\square 1 \quad f(x) \quad \square \square \square \quad (0, \pi) \quad \square \square \square \square \square \square \square \square$$

$$\square 2 \quad f(x) \quad \square \square \square \square 2 \quad \square \square \square \square$$

$$\square \square \square \square \square \square \square \square 1 \square \square \square \quad f(x) = \ln x - x + 2 \sin x \quad \square \quad x \in (0, \pi) \quad \square$$

$$\therefore f(x) = \frac{1}{x} - 1 + 2 \cos x \quad \square$$

$$\square \quad g(x) = \frac{1}{x} - 1 + 2 \cos x \quad \square \quad x \in (0, \pi) \quad \square$$

$$\therefore g(x) = -\frac{1}{x^2} - 2\sin x < 0 \quad \therefore g(x) \text{ 在 } (0, \pi) \text{ 上恒成立}$$

$$\text{当 } x \rightarrow 0 \text{ 时 } g(x) \rightarrow +\infty \quad g\left(\frac{\pi}{2}\right) = \frac{2}{\pi} - 1 < 0$$

$$\therefore \text{存在 } x_0 \in (0, \frac{\pi}{2}) \text{ 使得 } g(x_0) = 0$$

$$\therefore \text{当 } x \in (0, x_0) \text{ 时 } g(x) > 0 \quad f'(x) > 0 \quad \text{当 } x \in (x_0, \pi) \text{ 时 } g(x) < 0 \quad f'(x) < 0 \quad f(x) \text{ 在 } (0, \pi) \text{ 上先增后减}$$

$$\therefore f(x) \text{ 在 } (0, \pi) \text{ 上的最大值为 } f(x_0)$$

$$\text{2. 证明 } f(x) \text{ 在 } (0, x_0) \text{ 上恒成立}$$

$$\therefore x_0 \text{ 满足 } f(x_0) = 0 \quad x_0 \in (0, \frac{\pi}{2})$$

$$\therefore f(x_0) > f\left(\frac{\pi}{2}\right) = \ln\frac{\pi}{2} - \frac{\pi}{2} + 2 = \ln\frac{\pi}{2} + \frac{4-\pi}{2} > 0$$

$$\text{当 } x \rightarrow 0 \text{ 时 } f(x) \rightarrow -\infty \quad f(\pi) = \ln\pi - \pi < 0$$

$$\therefore \text{存在 } x_1 \in (0, x_0) \text{ 使得 } f(x_1) = 0 \quad \text{存在 } x_2 \in (x_0, \pi) \text{ 使得 } f(x_2) = 0$$

$$\text{当 } x \in (\pi, +\infty) \text{ 时 } h(x) = \ln x - x \quad h'(x) = \frac{1}{x} - 1 = \frac{1-x}{x} < 0$$

$$\therefore h(x) \text{ 在 } (\pi, +\infty) \text{ 上恒成立 } h(x) < h(\pi) = \ln\pi - \pi < 0$$

$$\textcircled{1} \text{ 当 } x \in (\pi, 2\pi) \text{ 时 } \sin x < 0 \quad \therefore \text{当 } x \in (\pi, 2\pi) \text{ 时 } f(x) < 0$$

$$\textcircled{2} \text{ 当 } x \in (2\pi, +\infty) \text{ 时 } h(x) < h(2\pi) = \ln(2\pi) - 2\pi < \ln e^3 - 2\pi < -2 - 2\pi < -2 - 2\sin x, 2 \quad \therefore \text{当 } x \in (2\pi, +\infty) \text{ 时 } f(x) < 0$$

证明

$$\therefore \text{当 } x \in (\pi, +\infty) \text{ 时 } f(x) = \ln x - x + 2\sin x < 0 \quad \therefore f(x) \text{ 在 } (\pi, +\infty) \text{ 上恒成立}$$

$$\therefore f(x) \text{ 在 } (0, 2\pi) \text{ 上恒成立}$$

$$3. \text{ 证明 } f(x) = \sin x - e^{x-2} \text{ 在 } (0, 2) \text{ 上恒成立}$$

1 $f(x)$ $(0, \frac{\pi}{2})$

2 $f(x)$ $(0, +\infty)$ 2

$f(x) = \sin x \cdot e^{x^2}$ $f'(x) = \cos x \cdot e^{x^2}$

$g(x) = f'(x)$ $g(x) = -\sin x \cdot e^{x^2}$ $x \in (0, \frac{\pi}{2})$ $g(x) < 0$ $g(x)$ $f(x)$ $(0, \frac{\pi}{2})$

$f(0) = 1 \cdot \frac{1}{e} > 0$ $(\frac{\pi}{2}) = -e^{\frac{\pi^2}{2}} < 0$ $f(x)$ $f(x)$ $(0, \frac{\pi}{2})$ α

$x \in (0, \alpha)$ $f(x) > 0$ $x \in (\alpha, \frac{\pi}{2})$ $f(x) < 0$

$f(x)$ $(0, \alpha)$ $(\alpha, \frac{\pi}{2})$

$f(x)$ $(0, \frac{\pi}{2})$

2 $f(x) = \sin x \cdot e^{x^2}$ $f'(x) = \cos x \cdot e^{x^2}$

$g(x) = f'(x)$ $g(x) = -\sin x \cdot e^{x^2}$ $x \in (0, \pi)$ $g(x) < 0$ $g(x)$ $(0, \pi)$

1 $f(x)$ $(0, \alpha)$ $(\alpha, \frac{\pi}{2})$

$f(0) = -e^2 < 0$ $f(\frac{\pi}{2}) > 0$ $f(\alpha) > (\frac{\pi}{2}) > 0$

$f(x)$ $x \in (0, \alpha)$ $f(x) = 0$

$x \in [\frac{\pi}{2}, \pi]$ $f(x) < 0$ $f(x)$ $[\frac{\pi}{2}, \pi]$

$f(\pi) = -e^{\pi^2} < 0$ $f(\frac{\pi}{2}) > 0$ $f(x)$

$x_2 \in (\frac{\pi}{2}, \pi)$ $f(x_2) = 0$

$x \in (\pi, +\infty)$ $e^{x^2} > 1$ $\sin x, 1$ $f(x) < 0$ $f(x)$ $(\pi, +\infty)$

□□□ $f(x)$ □□□□□□□□

4□□□□□ $f(x) = \cos x + \frac{1}{4}x^2 - 1$

□1□□□□ $f(x), 0$ $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

□2□□□ $y = f(x)$ □□□□□□□□□□□□□□

□□□□□□□1□□□□□□ $f(x) = \cos x + \frac{1}{4}x^2 + 1$ $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ □

□□ $f(x)$ □□□□□

□□□ $g(x) = \cos x + \frac{1}{4}x^2 + 1$ $x \in [0, \frac{\pi}{2}]$ □

□□ $g'(x) = -\sin x + \frac{1}{2}x$ $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ □

□□ $g'(x) = -\cos x + \frac{1}{2}$ □

□ $x \in [0, \frac{\pi}{3}]$ □□ $g'(x) < 0$ □□ $x \in (\frac{\pi}{3}, \frac{\pi}{2}]$ □□ $g'(x) < 0$ □

□□□ $g(x)$ □ $[0, \frac{\pi}{3}]$ □□□□□□ $[\frac{\pi}{3}, \frac{\pi}{2}]$ □□□□□□

□ $g(0) = 0$ □ $g(\frac{\pi}{2}) = \frac{\pi}{4} - 1 < 0$ □

□□ $g(x), 0$ □

□ $g(x)$ □ $[0, \frac{\pi}{2}]$ □□□□□

□ $g(x)_{\max} = g(0) = 0$ □

□ $g(x), 0$ □

□□ $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ □□ $f(x), 0$ □

□2□④□□1□□□□ $x \in [0, \frac{\pi}{2}]$ □□□□ $f(x)$ □□□□ 1 □□□□ $x = 0$ □

② $x \in [3, +\infty)$ $f'(x) = -\sin x + \frac{1}{2}x > 0$

$f(x)$ $[3, +\infty)$ $\square\square\square\square$

$f'(x) = -\cos 3 + \frac{5}{4} > 0$

$y = f(x)$ $[3, +\infty)$ $\square\square\square\square$

③ $x \in (\frac{\pi}{2}, 3)$ $\square\square$

$f'(x) = -\cos x + \frac{1}{2} > 0$

$f(x) = -\sin x + \frac{1}{2}x$ $\square\square\square\square$

$f(\frac{\pi}{2}) = \frac{\pi}{4} - 1 < 0$ $f'(3) > 0$ \square

x_0 $f(x_0) = 0$ \square

$\frac{\pi}{2} < x < x_0$ $f(x) < 0$ $x_0 < x < 3$ $f(x) > 0$ \square

$f(x)$ $(\frac{\pi}{2}, x_0)$ $\square\square\square\square\square$ $(x_0, 3)$ $\square\square\square\square$

$f(\frac{\pi}{2}) = \frac{\pi^2}{16} - 1 < 0$ $f'(3) = \frac{5}{4} + \cos 3 > 0$ \square

$f(x)$ $(\frac{\pi}{2}, 3)$ $\square\square 1$ $\square\square\square$

$y = f(x)$ R $\square\square\square\square$

①②③ $\square\square$

$[-3, -\frac{\pi}{2})$ $\square 1$ $\square\square\square\square$ $(-\infty, -3)$ $\square\square\square\square$ $[-\frac{\pi}{2}, 0)$ $\square\square\square$

$f(x)$ R $\square\square 3$ $\square\square\square$

5 $f(x) = \ln x - \sin x + ax (a > 0)$ \square

$$\therefore \square \square \quad y = \frac{1}{x} + a \quad (a > 0) \quad \square \square \quad y = \cos x \quad x \in \left(\frac{3\pi}{2}, 2\pi\right) \quad \square \square \square \square \square \square \square \square$$

$$\therefore \frac{1}{2\pi} + a < \cos 2\pi = 1 \quad \square \square \quad a < 1 - \frac{1}{2\pi} \quad \square$$

$$\square \square \quad a \quad \square \square \square \square \square \square \quad \left(0, 1 - \frac{1}{2\pi}\right) \quad \square$$

$$6 \square \square \square \square \quad f(x) = e^x - ax \quad (a \in \mathbb{R}) \quad \square$$

$$\square 1 \square \square \square \square \quad f(x) \quad \square \square \square \square \square$$

$$\square 2 \square \square \quad a = 2 \square \square \square \square \quad g(x) = f(x) - \cos x \quad x \in \left(-\frac{\pi}{2}, +\infty\right) \quad \square \square \square \square \square \square \square$$

$$\square \square \square \square \square \square 1 \square \quad f(x) = e^x - ax \quad \square \square \square \square \square \quad \mathbb{R} \square \quad f(x) = e^x - a \quad \square$$

$$\textcircled{1} \square \quad a, 0 \quad \square \square \square \square \quad f(x) > 0 \quad \square \square \square \quad f(x) \quad \square \quad \mathbb{R} \quad \square \square \square \square \square \square$$

$$\textcircled{2} \square \quad a > 0 \quad \square \square \square \quad f(x) > 0 \quad \square \quad x > \ln a \quad \square \square \quad f(x) < 0 \quad \square \quad x < \ln a \quad \square$$

$$\square \square \quad f(x) \quad \square \quad (-\infty, \ln a) \quad \square \square \square \square \square \square \quad (\ln a, +\infty) \quad \square \square \square \square \square \square$$

$$\square \square \square \square \square \quad a, 0 \quad \square \square \quad f(x) \quad \square \quad \mathbb{R} \quad \square \square \square \square \square \square \quad a > 0 \quad \square \square \quad f(x) \quad \square \quad (-\infty, \ln a) \quad \square \square \square \square \square \square \quad (\ln a, +\infty) \quad \square \square \square \square \square \square$$

$$\square 2 \square \square \quad a = 2 \square \square \quad g(x) = e^x - 2x - \cos x \quad x \in \left(-\frac{\pi}{2}, +\infty\right) \quad \square \quad g(x) = e^x + \sin x - 2 \quad \square$$

$$\textcircled{1} \square \quad x \in \left(-\frac{\pi}{2}, 0\right) \quad \square \square \square \square \quad g(x) = (e^x - 1) + (\sin x - 1) < 0 \quad \square$$

$$\square \square \quad g(x) \quad \square \quad \left(-\frac{\pi}{2}, 0\right) \quad \square \square \square \square \square$$

$$\square \square \quad g(x) > g(0) = 0 \quad \square$$

$$\square \square \quad g(x) \quad \square \quad \left(-\frac{\pi}{2}, 0\right) \quad \square \square \square \square \square$$

$$\textcircled{2} \square \quad x \in \left[0, \frac{\pi}{2}\right] \quad \square \square \square \square \quad g(x) \quad \square \square \square \square \square \square \quad g(0) = -1 < 0, \quad g\left(\frac{\pi}{2}\right) = e^{\frac{\pi}{2}} - 1 > 0 \quad \square$$

$$\text{因为 } x_0 \in (0, \frac{\pi}{2}) \text{ 所以 } g'(x_0) = 0$$

$$\text{当 } x \in (0, x_0) \text{ 时 } g'(x) < 0 \text{ 当 } x \in (x_0, \frac{\pi}{2}) \text{ 时 } g'(x) > 0$$

$$\text{所以 } g(x) \text{ 在 } [0, x_0) \text{ 上单调递减, 在 } (x_0, \frac{\pi}{2}] \text{ 上单调递增, } g(0) = 0 \text{ 所以 } g(x_0) < 0$$

$$\text{因为 } g(\frac{\pi}{2}) = e^{\frac{\pi}{2}} - \pi > 0 \text{ 所以 } g(x_0) \cdot g(\frac{\pi}{2}) < 0$$

$$\text{所以 } g(x) \text{ 在 } (x_0, \frac{\pi}{2}) \text{ 上有唯一零点}$$

$$\text{所以 } g(x) \text{ 在 } [0, \frac{\pi}{2}] \text{ 上有两个零点}$$

$$\textcircled{3} \text{ 当 } x \in (\frac{\pi}{2}, +\infty) \text{ 时 } g'(x) = e^x + \sin x - 2 > e^{\frac{\pi}{2}} - 3 > 0$$

$$\text{所以 } g(x) \text{ 在 } (\frac{\pi}{2}, +\infty) \text{ 上单调递增}$$

$$\text{因为 } g(\frac{\pi}{2}) > 0 \text{ 所以 } g(x) \text{ 在 } (\frac{\pi}{2}, +\infty) \text{ 上没有零点}$$

$$\text{所以 } g(x) \text{ 在 } (-\frac{\pi}{2}, +\infty) \text{ 上有两个零点}$$

$$7 \text{ 因为 } y = e^x - 2\sin x - 2x\cos x \text{ 在 } (-\pi, -\frac{\pi}{2}) \text{ 上单调递增}$$

$$\text{因为 } f(x) = \frac{e^x}{x} - 2\sin x \text{ 在 } (-\pi, 0) \text{ 上单调递增, 所以 } x_0 \text{ 满足 } 0 < f(x_0) < 2$$

$$\text{因为 } y' = e^x - 2\cos x - 2(\cos x - x\sin x) = e^x + 2x\sin x - 4\cos x \text{ 在 } (-\pi, -\frac{\pi}{2}) \text{ 上}$$

$$\text{因为 } e^x > 0, 2x\sin x > 0, -4\cos x > 0 \text{ 所以 } y' > 0$$

$$\text{所以 } y \text{ 单调递增}$$

$$\text{因为 } f(x) = \frac{e^x(x-1) - 2x^2\cos x}{x^2}$$

$$g(x) = e^x(x-1) - 2x^2 \cos x \quad g'(x) = x(e^x + 2x \sin x - 4 \cos x)$$

$$x \in (-\pi, -\frac{\pi}{2}) \quad 1 \quad g'(x) < 0 \quad g(x) \quad$$

$$g(-\frac{\pi}{2}) = e^{-\frac{\pi}{2}}(-\frac{\pi}{2} - 1) < 0 \quad g(-\pi) = 8 - e^\pi(1 + \pi) > 0$$

$$x_0 \in (-\pi, -\frac{\pi}{2}) \quad g(x_0) = 0$$

$$x \in (-\pi, x_0) \quad g(x) > 0 \quad f(x) \quad$$

$$x \in (x_0, -\frac{\pi}{2}) \quad g(x) < 0 \quad f(x) \quad$$

$$x \in (-\frac{\pi}{2}, 0) \quad f(x) = \frac{e^x(x-1)}{x^2} - 2 \cos x < 0 \quad f(x) \quad$$

$$f(x) \quad (x_0, 0) \quad$$

$$f(x) \quad x_0$$

$$f(x_0) > f(-\frac{\pi}{2}) = \frac{e^{-\frac{\pi}{2}}}{-\frac{\pi}{2}} + 2 = -\frac{1}{\frac{\pi}{2}e^{\frac{\pi}{2}}} + 2 > 0$$

$$f(x_0) = \frac{e^{x_0}}{x_0} - 2 \sin x_0 \quad x_0 \in (-\pi, -\frac{\pi}{2}) \quad \frac{e^{x_0}}{x_0} \in (-1, 0) \quad 0 < -2 \sin x_0 < 2$$

$$f(x_0) < 2$$

8 $f(x) = \tan x \cdot \sin x$ $g(x) = x \cdot \sin x$ $x \in (0, \frac{\pi}{2})$

1 $f(x) - g(x) = x$ $(0, \frac{\pi}{2})$

2 $x \in (0, \frac{\pi}{2})$ $f(x) \cdot g(x)$ a

1 $h(x) = f(x) - g(x) = x$ $h(x) = \tan x - 2x$

$h(x) = \frac{1}{\cos^2 x} - 2 = \frac{1 - 2\cos^2 x}{\cos^2 x} = \frac{(1 + \sqrt{2}\cos x)(1 - \sqrt{2}\cos x)}{\cos^2 x}$

$x \in (0, \frac{\pi}{4})$ $\cos x > \frac{\sqrt{2}}{2}$ $h(x) < 0$ $x \in (\frac{\pi}{4}, \frac{\pi}{2})$ $h(x) > 0$

$h(x) = \tan x - 2x$ $x \in (0, \frac{\pi}{4})$ $x \in (\frac{\pi}{4}, \frac{\pi}{2})$

$h(\frac{\pi}{4}) < 0$ $h(\frac{5\pi}{12}) = \tan(\frac{5\pi}{12}) - \frac{5\pi}{6} = 2 + \sqrt{3} - \frac{5\pi}{6} > 2 + 1.7 - 2.5 > 0$

$h(x) = \tan x - 2x$ $x \in (0, \frac{\pi}{4})$ $x \in (\frac{\pi}{4}, \frac{\pi}{2})$

2 $\varphi(x) = f(x) - g(x) = \tan x \cdot \sin x - x \cdot \sin x$

$\varphi'(x) = \frac{1}{\cos^2 x} - \cos x - x(1 - \cos x) = \frac{(1 - \cos^3 x)}{\cos^2 x} - x(1 - \cos x)$

① $a, 0$ $x \in (0, \frac{\pi}{2})$ $\varphi'(x) > 0$

$\varphi(x)$ $(0, \frac{\pi}{2})$ $\varphi(0) = 0$ $\varphi(x) > 0$

② $1 < a, 3$ $\varphi'(x) = \frac{2\sin x}{\cos^3 x} + \sin x - a\sin x = \sin x(\frac{2}{\cos^3 x} + 1 - a)$

$x \in (0, \frac{\pi}{2})$ $\cos x \in (0, 1)$ $\frac{2}{\cos^3 x} + 1 \in (3, +\infty)$

$$1 < a, 3 \varphi''(x) > 0$$

$$\varphi(x) \quad x \in (0, \frac{\pi}{2}) \quad \varphi'(0) = 0$$

$$\varphi'(x) > 0 \quad \varphi(x) \quad x \in (0, \frac{\pi}{2}) \quad \varphi(0) = 0 \quad \varphi(x) > 0 \quad x \in (0, \frac{\pi}{2})$$

$$\textcircled{3} \quad a > 3 \quad \varphi''(x) = \sin x \left(\frac{2 - (a-1)\cos^2 x}{\cos^3 x} \right) = 0$$

$$\cos x = \sqrt{\frac{2}{a-1}} \in (0, 1) \quad x_0$$

$$x \in (0, x_0) \quad \varphi''(x) < 0 \quad x \in (x_0, \frac{\pi}{2}) \quad \varphi''(x) > 0$$

$$x \in (0, x_0) \quad \varphi'(x) \quad x \in (x_0, \frac{\pi}{2}) \quad \varphi'(x)$$

$$\varphi'(0) = 0 \quad \varphi'(x) < 0 \quad x \in (0, x_0)$$

$$\varphi(x) \quad x \in (0, x_0) \quad \varphi(0) = 0 \quad \varphi(x_0) < 0$$

$$a, 3 \quad a \quad 3$$

$$\varphi(x) = f(x) - ag(x) = \tan x - \sin x - a(x - \sin x)$$

$$\varphi'(x) = \frac{1}{\cos^2 x} - \cos x - a(1 - \cos x) = \frac{(1 - \cos^3 x)}{\cos^2 x} - a(1 - \cos x)$$

$$\varphi'(x) > 0 \quad a < \frac{1 + \cos x + \cos^2 x}{\cos^2 x}$$

$$g(x) = \frac{1 + \cos x + \cos^2 x}{\cos^2 x} \quad x \in (0, \frac{\pi}{2})$$

$$\cos x = t \quad t \in (0, 1)$$

$$\square \quad x \in (0, x_0) \quad \square \square \quad f(x) > 0 \quad \square \quad f(x) \quad \square \square \square$$

$$\square \quad x \in (x_0, \pi) \quad \square \square \quad f(x) < 0 \quad \square \quad f(x) \quad \square \square \square$$

$$\therefore f(x) \quad \square \quad x_0 \quad \square \square \square \square \square \square \square \square \square \square \square$$

$$\textcircled{2} \quad \square \quad f(\pi) = 0 \quad \square \square \quad a \cdot 1 - \frac{1}{\pi} \quad \square \quad f(x) > 0 \quad \square$$

$$\therefore f(x) \quad \square \quad (0, \pi] \quad \square \square \square \square \quad f(x) \quad \square \square \square \square$$

$\square 3 \square \square \square 2 \square \square \square \square$

$$(i) \quad \square \quad a \cdot 1 - \frac{1}{\pi} \quad \square \square \square \square \square \square \square \square$$

$$f(x)_{\min} = f\left(\frac{\pi}{2}\right) = \ln \frac{\pi}{2} + \frac{\pi}{2} \left(1 - \frac{1}{\pi}\right) + 1 = \ln \frac{\pi}{2} + \frac{\pi}{2} + \frac{1}{2} > 0 \quad \square$$

$$\square \quad a \cdot 1 - \frac{1}{\pi} \quad \square \square \square \square \square \square \square \square$$

$$(ii) \quad \square \quad a < 1 - \frac{1}{\pi} \quad \square \square \quad x \in (0, x_0] \quad \square \square \quad f(x) \quad \square \square \square$$

$$x \in (x_0, \pi] \quad \square \square \quad f(x) \quad \square \square \square$$

$$\square \quad f(x_0) = 0 \quad \square \quad \frac{1}{x_0} + a + \cos x_0 = 0 \quad \square \square \square \quad a = -\frac{1}{x_0} - \cos x_0 \quad \square$$

$$\square \square \quad h(x) = -\frac{1}{x} - \cos x \quad \square \square \quad h(x) = \frac{1}{x^2} + \sin x > 0 \quad \square \square \quad a \square \square \quad x_0 \quad \square \square \square$$

$$\textcircled{1} \quad \square \quad x_0 \in (0, \frac{\pi}{2}] \quad \square \square \square \quad a \in (-\infty, -\frac{2}{\pi}] \quad \square$$

$$\square \quad f\left(\frac{\pi}{2}\right) = (\pi) > 0 \quad \square \quad a < -\frac{2}{\pi} \left(1 + \ln \frac{\pi}{2}\right) \quad \square \quad a > -\frac{\ln \pi}{\pi} \quad \square$$

$$\therefore a < -\frac{2}{\pi} \left(1 + \ln \frac{\pi}{2}\right) \quad \square \square \square \square \square$$

$$f\left(\frac{\pi}{2}\right) = (\pi) < 0 \quad \square \quad -\frac{2}{\pi} \left(1 + \ln \frac{\pi}{2}\right) < a < -\frac{\ln \pi}{\pi} \quad \square$$

$$\therefore -\frac{2}{\pi} \left(1 + \ln \frac{\pi}{2}\right) < a, -\frac{2}{\pi} \quad \square \square \quad 1 \quad \square \square \square \square$$

$$a = -\frac{2}{\pi}(1 + \ln \frac{\pi}{2}) \quad f(\frac{\pi}{2}) = 0 \quad f(\pi) \neq 0 \quad \text{1}$$

$$a < -\frac{2}{\pi}(1 + \ln \frac{\pi}{2}) \quad -\frac{2}{\pi}(1 + \ln \frac{\pi}{2}), \quad a, -\frac{2}{\pi} \quad \text{1}$$

$$\textcircled{2} \quad x_0 \in (\frac{\pi}{2}, \pi] \quad a \in (-\frac{2}{\pi}, 1 - \frac{1}{\pi}]$$

$$f(\frac{\pi}{2}) = \ln \frac{\pi}{2} + \frac{\pi}{2} a + 1 > 0 \quad f(\pi) = \ln \pi + \pi a$$

$$\therefore f(x)_{\min} = f(x_0) = \ln x_0 + ax_0 + \sin x = \ln x_0 + \sin x_0 - x_0 \cos x_0 - 1$$

$$m(x) = \ln x + \sin x - x \cos x - 1 \quad m'(x) = \frac{1}{x} + x \sin x > 0$$

$$f(x)_{\min} > m(\frac{\pi}{2}) = \ln \frac{\pi}{2} > 0$$

$$f(\pi) > 0 \quad a > -\frac{\ln \pi}{\pi} \quad -\frac{\ln \pi}{\pi} < a < 1 - \frac{1}{\pi}$$

$$f(\pi), 0 \quad a, -\frac{\ln \pi}{\pi} \quad -\frac{2}{\pi} < a, -\frac{\ln \pi}{\pi} \quad \text{1}$$

$$a \in (-\infty, -\frac{2}{\pi}(1 + \ln \frac{\pi}{2}) \cup (-\frac{\ln \pi}{\pi}, +\infty)$$

$$a \in [-\frac{2}{\pi}(1 + \ln \frac{\pi}{2}) - \frac{\ln \pi}{\pi}] \quad \text{1}$$

$$10 \quad f(x) = (x-1) - (x+2)\sin x$$

$$1 \quad x \in [\frac{\pi}{2}, \pi] \quad y = f(x)$$

$$2 \quad x \in [0, 2\pi] \quad y = f(x)$$

$$f(x) = (x-1) - (x+2)\sin x \quad x \in [\frac{\pi}{2}, \pi]$$

$$f(x) = 1 - \sin x - (x+2)\cos x$$

$$\frac{\pi}{2}, x, \pi \quad \cos x, 0 \quad \sin x, 1$$

$$\therefore f(x) \leq 0 \quad f(x) \in [\frac{\pi}{2}, \pi]$$

$$\square f(\frac{\pi}{2}) = -3 < 0 \quad \square f(\pi) = \pi - 1 > 0 \quad \square$$

$$\therefore \square \square f(x) \square [\frac{\pi}{2}, \pi] \square \square \square \square \square \square$$

$$\square 2 \square \square \square f(x) = (x-1) - (x+2)\sin x \quad \square x \in [0, 2\pi] \quad \square$$

$$\square f'(x) = 1 - \sin x - (x+2)\cos x \quad \square$$

$$\square h(x) = 1 - \sin x - (x+2)\cos x \quad \square h'(x) = -2\cos x + (x+2)\sin x \quad \square$$

$$\textcircled{1} \square \square 0, x, \frac{\pi}{4} \square \square \cos x \cdot \frac{\sqrt{2}}{2} - 1 - 2\cos x < 1 - 2 \times \frac{\sqrt{2}}{2} = 1 - \sqrt{2} < 0 \quad \square$$

$$\therefore f(x) = 1 - \sin x - (x+2)\cos x = (1 - 2\cos x) - \sin x - x\cos x < 0 \quad \square$$

$$\therefore \square \square f(x) \square [0, \frac{\pi}{4}] \square \square \square \square \square \square$$

$$\textcircled{2} \square \square \frac{\pi}{4} < x < \pi \quad \square \square h(\frac{\pi}{2}) = 0 \quad \square$$

$$\square \square \frac{\pi}{2} < x < \pi \quad \square \square \cos x < 0 \quad \square \therefore h(x) = -2\cos x + (x+2)\sin x > 0 \quad \square$$

$$\therefore h(x) \square [\frac{\pi}{2}, \pi] \square \square \square \square h(x) > h(\frac{\pi}{2}) = 0 \quad \square f(x) > 0 \quad \square$$

$$\square \square \frac{\pi}{4} < x < \frac{\pi}{2} \quad \square \square \sin x > \cos x \quad \square$$

$$\therefore h(x) = -2\cos x + (x+2)\sin x = 2(\sin x - \cos x) + x\sin x > 0 \quad \square$$

$$\therefore h(x) \square (\frac{\pi}{4}, \frac{\pi}{2}) \square \square \square h(x) < h(\frac{\pi}{2}) = 0 \quad \square f(x) < 0 \quad \square$$

$$\therefore \frac{\pi}{2} \square f(x) \square (\frac{\pi}{4}, \pi) \square \square \square \square \square \square$$

$$\textcircled{3} \square \square \pi < x, \frac{3\pi}{2} \square \square \sin x < 0 \quad \square \cos x, 0 \square \square f'(x) > 0 \quad \square f(x) \square \square \square \square \square$$

$$\textcircled{4} \square \square \frac{3\pi}{2} < x, 2\pi \square \square \cos x > 0 \quad \square \sin x < 0 \quad \square$$

$$\therefore h(x) = -2\cos x + (x+2)\sin x < 0$$

$$\therefore h(x) \Big|_{(\frac{3\tau}{2}, 2\tau)} = h(\frac{3\tau}{2}) = 2 > 0 \quad h(2\tau) = -2\tau - 1 < 0$$

$$\therefore h(x) \Big|_{(\frac{3\tau}{2}, 2\tau)} \quad x_2$$

$$\frac{3\tau}{2} < x < x_2 \quad f(x) > 0 \quad x_2 < x < 2\tau \quad f(x) < 0$$

$$x = x_2 \quad f(x)$$

$$f(x) \quad 2$$

$$11 \quad f(x) = (1-a-x)\sin x - (1+a+x)\cos x \quad x \in [0, \pi] \quad a \in \mathbb{R}$$

$$1 \quad f(x) \Big|_{(\frac{\pi}{2}, f(\frac{\pi}{2}))} \quad \frac{\pi}{2} + 1 \quad a$$

$$2 \quad x \in [0, \pi] \quad f(x) \dots 0 \quad a$$

$$1 \quad f(x) = (1-a-x)\sin x - (1+a+x)\cos x$$

$$\therefore f(x) = (x+a)(\sin x - \cos x)$$

$$\quad f(x) \Big|_{(\frac{\pi}{2}, f(\frac{\pi}{2}))} \quad \frac{\pi}{2} + 1$$

$$\therefore f(\frac{\pi}{2}) = \frac{\pi}{2} + a = \frac{\pi}{2} + 1 \quad a = 1$$

$$2 \quad 1 \quad f(x) = (x+a)(\sin x - \cos x) \quad x \in [0, \pi]$$

$$f(x) = 0 \quad x = -a \quad x_2 = \frac{\pi}{4}$$

$$① \quad a > 0 \quad x + a > 0 \quad x \in [0, \frac{\pi}{4}] \quad \sin x - \cos x < 0$$

$$f(x) \geq 0 \quad f(x)$$

$$x \in [\frac{\pi}{4}, \pi] \quad \sin x - \cos x \geq 0 \quad f(x) \geq 0 \quad f(x)$$

$$\forall x \in [0, \pi] \quad f(x) \geq 0$$

$$f(x)_{\min} = f\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \left(1 - a - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{2} \left(1 + a + \frac{\pi}{4}\right) \geq 0 \quad a \leq -\frac{\pi}{4}$$

$$\textcircled{2} \quad -\frac{\pi}{4} < a < 0 \quad \forall x \in [0, -a) \quad x + a < 0 \quad \sin x - \cos x < 0$$

$$f(x) > 0 \quad f(x)$$

$$\forall x \in (-a, \frac{\pi}{4}) \quad x + a > 0 \quad \sin x - \cos x < 0 \quad f(x) < 0 \quad f(x)$$

$$\forall x \in (\frac{\pi}{4}, \pi] \quad x + a > 0 \quad \sin x - \cos x > 0 \quad f(x) > 0 \quad f(x)$$

$$\forall x \in [0, \pi] \quad f(x) \geq 0 \quad \begin{cases} f(0) \geq 0 \\ f(\frac{\pi}{4}) \geq 0 \end{cases} \quad a \geq -1$$

$$\textcircled{3} \quad -\pi \leq a \leq -\frac{\pi}{4} \quad \textcircled{2} \quad f(x) \geq 0 \quad [0, \frac{\pi}{4}) \quad (\frac{\pi}{4}, -a) \quad (-a, \pi]$$

$$\forall x \in [0, \pi] \quad f(x) \geq 0 \quad \begin{cases} f(0) \geq 0 \\ f(-a) \geq 0 \end{cases} \quad -\pi \leq a \leq -1 \quad a \in [-\pi, -1]$$

$$\textcircled{4} \quad a < -\pi \quad f(x) \geq 0 \quad [0, \frac{\pi}{4}) \quad (\frac{\pi}{4}, \pi]$$

$$\forall x \in [0, \pi] \quad f(x) \geq 0 \quad \begin{cases} f(\pi) \geq 0 \\ f(0) \geq 0 \end{cases} \quad -\pi \leq a \leq -1$$

$$a \in [-\pi - 1, -\pi]$$

$$a \in [-\pi - 1, -1]$$

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